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STRESS INTENSITY FACTORS AND DISPLACEMENTS FOR ARC BEND SAMPLES USING COLLOCATION

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Stress intensity factors, crack mouth opening displacements, and load-line displacements were determined for the arc bend-chord support specimen using boundary collocation. Several inner radius (r_1) to outer radius (r_2) ratios were studied at each of two span (S) to width (W) ratios. The actual solution matrix is for S/W = 3, r_1/r_2 = 0.8, 0.6, 0.4, and for S/W = 4, r_1/r_2 = 0.8, 0.7, 0.6. Also, the three-point bend single-edge specimen (SE(B)) at both S/W = 3 and S/W = 4 was studied to examine the limiting behavior of the

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20. ABSTRACT (CONT'D)

generated solutions. For all of the S/W = 3 cases, solutions were obtained for crack length to width ratios (a/W) from 0.2 to 0.6, and in the S/W = 4 cases, a/W was varied from 0.2 to 0.5. The solutions were obtained by the linear superposition of a pure bending stress, a pure shear stress, and a uniform normal stress on an annular segment. The magnitudes of the individual components depended upon the size of the annular segment and the S/W ratio. Wide range expressions were fit to the numerical solutions to make them applicable over a wider range of testing conditions and for inclusion in future revisions of E-399 on Plane-Strain Fracture Toughness Testing.

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INTRODUCTION

Underwood et al. (ref 1) have suggested that the arc bend-chord support sample (AB(C)) shown in Figure 1 would be a good addition to the presently available fracture toughness samples. In order to begin the standardization process, an accurate K solution is the minimum requirement. Additionally, crack mouth opening displacements and load-line displacement solutions would also be useful in future utilization of this specimen for fatigue crack propagation and J_{Ic} testing, for example. The previous study (ref 1) included these solutions generated by finite elements. This report presents the results of the analysis of the AB(C) specimen using boundary value collocation.

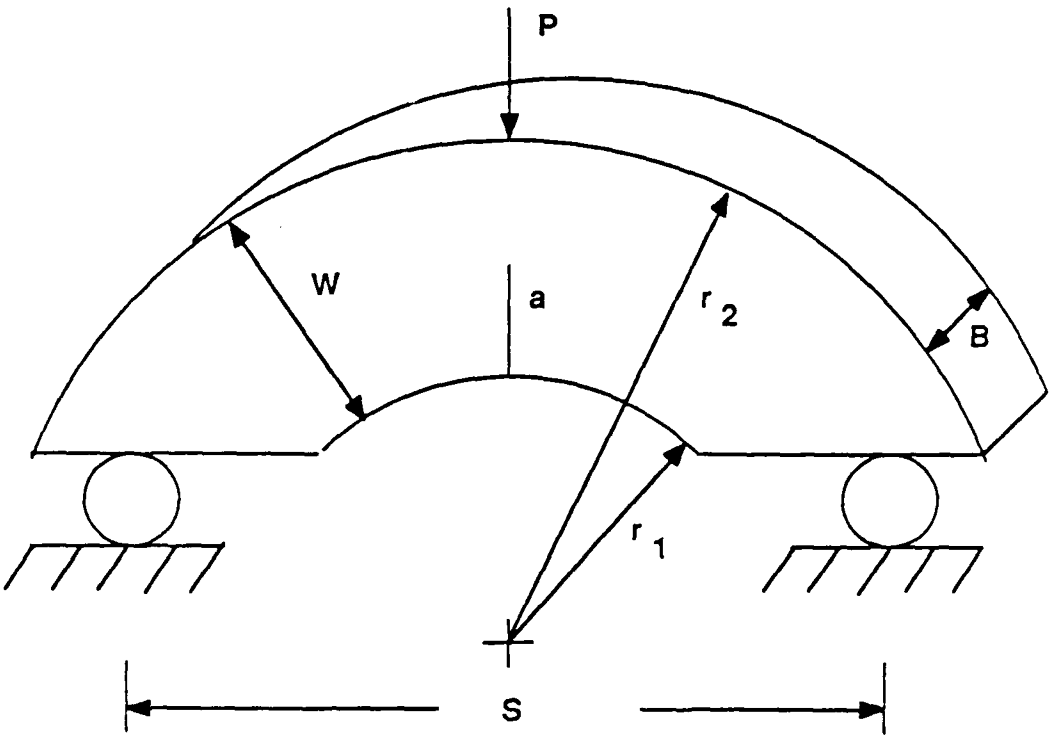

Figure 1. The arc bend-chord support specimen.

References are listed at the end of this report.

A wide variety of solutions were generated in at attempt to determine the effects of the three primary variables of the specimen: the crack length effect (a/W), the radius ratio effect (r_1/r_2), and the effect of span (S/W). It was decided to limit consideration of one of these variables (S/W) to just two values, namely $S/W = 3$ and $S/W = 4$. These values were chosen because the present standard bending specimen has $S/W = 4$. However, allowing only $S/W = 4$ limits the AB(C) sample to relatively thin-walled cylinders ($r_1/r_2 > \sim 0.6$). To include thicker-walled cylinders a shorter relative span must be allowed, thus $S/W = 3$ was also studied. Cylinders with radius ratios (r_1/r_2) greater than about 0.4 can be tested in this configuration. Three different radius ratios were studied for each of the two span ratio conditions, and for each specimen geometry, several crack lengths were studied. Finally, wide range expressions were fit to the numerical data to interpolate the solution between the specific conditions studied.

METHOD OF SOLUTION

The problem was solved using plane elastostatic boundary collocation of a homogeneous isotropic body. The method is well-documented in other sources (refs 2-4). In short, the general solution to a class of problems had to be found. Normally this solution is an infinite series solution that exactly satisfies some boundary condition of the particular problem considered. For crack problems, the general solution is the Williams' stress function (ref 5), which satisfies the traction-free condition of the crack surfaces. For the solution to a particular problem, the numerical value of the stress function and its derivative along the boundary of the body can be determined using known solutions for uncracked bodies. The coefficients in the general solution that

will make the truncated general solution satisfy the boundary conditions can then be obtained. This solution is normally obtained in the least squares sense by knowing the boundary conditions at an overdetermined number of boundary locations. The results reported herein were generated using the least squares scheme outlined by Hussain et al. (ref 4).

The particular problem shown in Figure 1 was solved by analyzing the annular ring segment shown in Figure 2. The value of θ_o was chosen large enough to include the entire AB(C) specimen. This was needed to determine the load-line displacements. Magnitudes of the various components were determined in order that they resolve to the reaction force at either roller support. These components are

$$\tau = \frac{P/BW}{2(\cos\theta_o + \tan\theta_o \sin\theta_o)} \tag{1}$$

$$\sigma = \frac{(P/BW)\tan\theta_o}{2(\cos\theta_o + \tan\theta_o \sin\theta_o)} \tag{2}$$

$$M = \frac{PW}{4}\left(\frac{S}{W} - \frac{(r_1+r_2)}{(r_2-r_1)}\sin\theta_o\right) \tag{3}$$

where τ is a uniform shearing stress and σ is a uniform normal stress. The other variables in the equations are shown in Figures 1 and 2.

Since the problem was symmetric about the plane of the crack, the stress function (ϕ) and its derivative with respect to the outward normal ($\partial\phi/\partial\eta$) only needed to be determined along the boundary ABCD. For the case of pure bending, a ring segment was developed by Gross and Srawley (ref 6). The other two loading conditions were not developed by Gross and Srawley, therefore they are developed and reported here. For the case of a uniform normal compressive stress σ acting as shown in Figure 2, the stress function and its derivative are

Along DC:

$$\phi = 0 \tag{4}$$

$$\frac{\partial\phi}{\partial\eta} = 0 \tag{5}$$

Along CB:

$$\phi = \sigma(r(r_1-r) - \frac{1}{2}(r_1^2-r^2)) \tag{6}$$

$$\frac{\partial\phi}{\partial\eta} = 0 \tag{7}$$

Along BA:

$$\phi = \sigma(r_2(r_1-r_2)\cos(\theta_o-\theta) - \frac{1}{2}(r_1^2-r_2^2)) \tag{8}$$

$$\frac{\partial\phi}{\partial\eta} = \sigma(r_1-r_2)\cos(\theta_o-\theta) \tag{9}$$

For the case of a uniform shearing stress τ that acts out from the inner radius, the stress function and its derivative are

Along DC:

$$\phi = 0 \tag{10}$$

$$\frac{\partial\phi}{\partial\eta} = 0 \tag{11}$$

Along CB:

$$\phi = 0 \tag{12}$$

$$\frac{\partial\phi}{\partial\eta} = \tau(r_1-r) \tag{13}$$

Along BA:

$$\phi = -\tau(r_1-r_2)r_2 \sin(\theta_o-\theta) \tag{14}$$

$$\frac{\partial\phi}{\partial\eta} = -\tau(r_1-r_2)\sin(\theta_o-\theta) \tag{15}$$

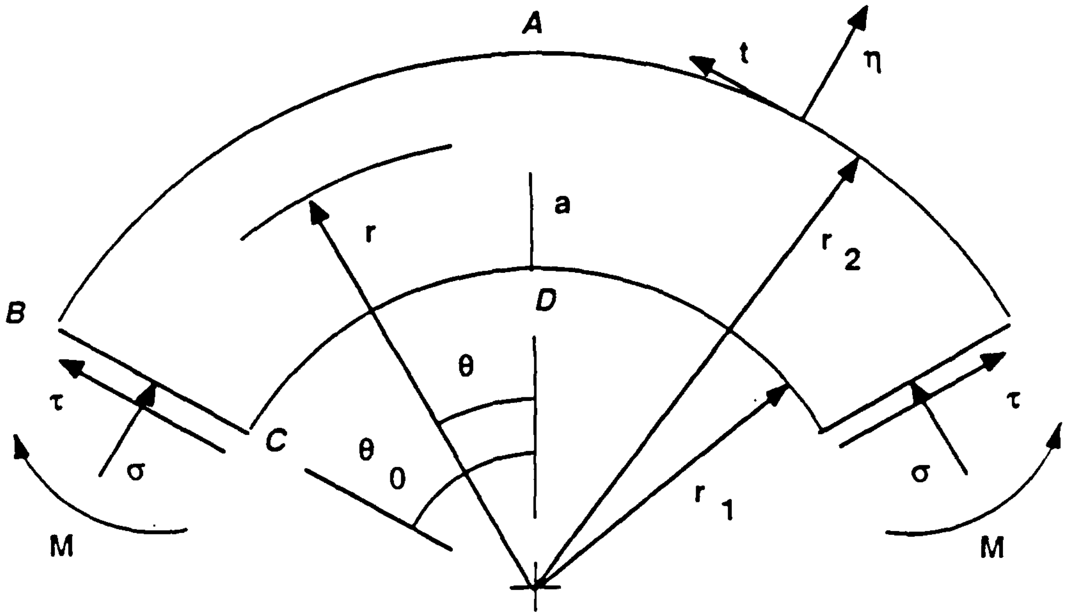

Figure 2. Annular segment used to analyze the arc bend-chord support specimen.

For the limiting case of a straight three-point bend sample, it is more convenient to derive the stress function using the geometry shown in Figure 3. In this case, the resultant shearing forces V should not be assumed to result in a uniform shear stress, but in the well-known parabolically varying distribution (ref 7). Using the solution in Reference 7 for a uniform rectangular section, the stress function and its derivative are

Along DC:

$$\phi = 0 \tag{16}$$

$$\frac{\partial \phi}{\partial \eta} = 0 \tag{17}$$

Along CB:

$$\phi = 0 \tag{18}$$

$$\frac{\partial \phi}{\partial \eta} = \frac{-6V}{W^3}\left(-\frac{1}{3}(a^3+x^3) + \frac{1}{2}(a^2-x^2)(2a-W) - a(a-W)(a+x)\right) \tag{19}$$

Along BA:

$$\phi = \frac{-(y-S)6V}{W^3} \left(\frac{1}{2}(2a-W)(a^2-(W-a)^2) - \frac{1}{3}(a^3+(W-a)^3) - W(a(a-W))^3\right) \tag{20}$$

$$\frac{\partial\phi}{\partial\eta} = 0 \tag{21}$$

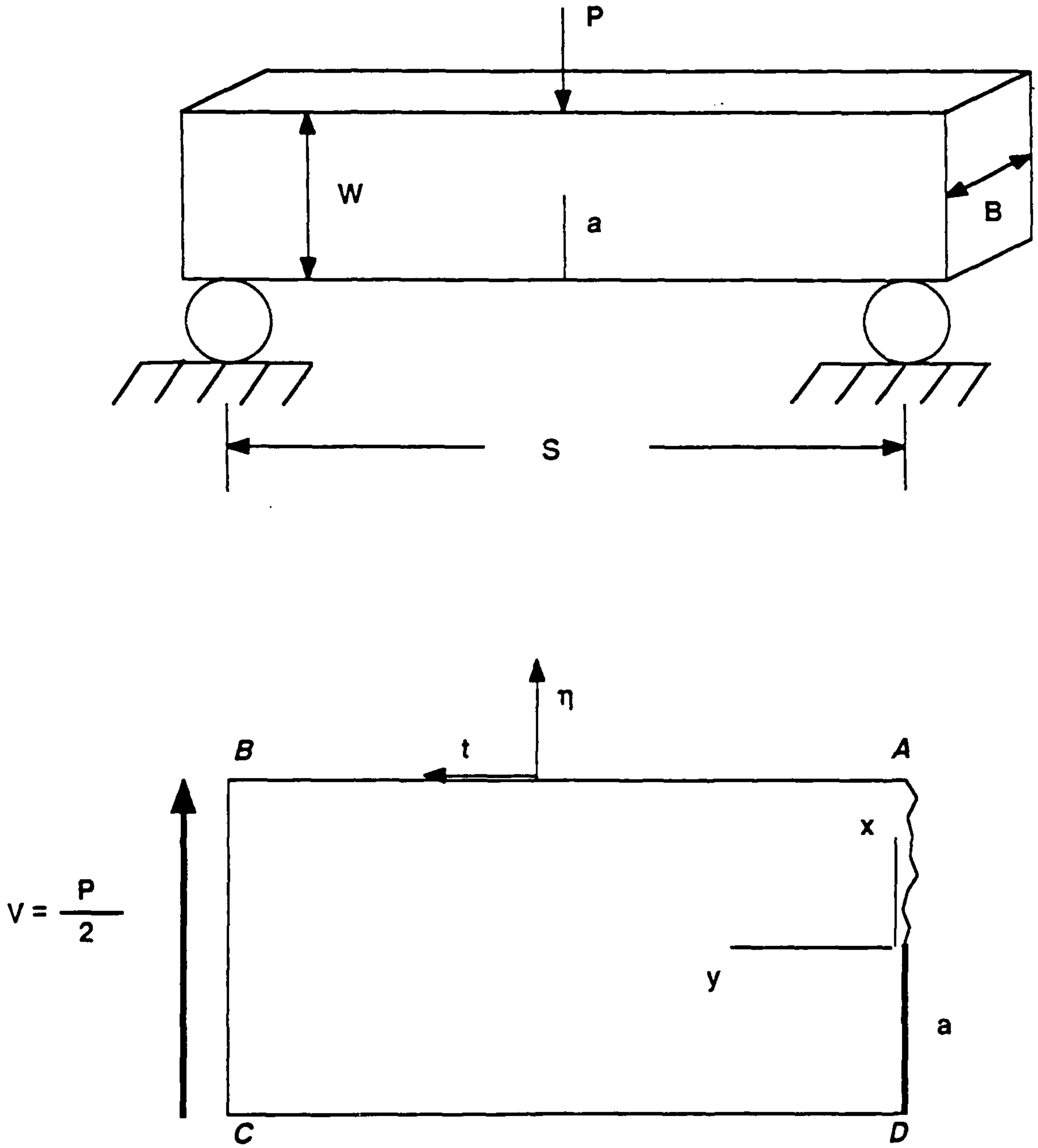

Figure 3. Geometry of straight rectangular bend specimen.

If the purpose of the analysis was to establish only K and crack mouth opening displacements, the choice of the parameter θ_0 was arbitrary as long as the resulting radial cut was far enough removed to satisfy St. Vennant's principle. Since load-line displacements were also desired, it was necessary to choose θ_0 such that the position of the roller support was somewhere contained in the region ABCD in Figure 2. This requirement was included since Williams' stress function (ref 5) is based on a coordinate system with its origin at the crack tip, thus the displacement of the crack tip is zero. In this coordinate system, the displacement at point A is the only displacement relative to the crack tip. The total load-point displacement was constructed as the displacement of the roller support relative to the crack tip plus the displacement of point A relative to the crack tip. This was accomplished by choosing θ_0 such that chord length $2r_2 \sin \theta_0$ was approximately 10 percent greater than the span S. With θ_0 at this value, load-line displacements were easily calculated using the coordinate system translation outlined above. It was necessary to make the chord length larger than S to eliminate some end effects that occurred in the curved geometry. No such end effects occurred for the straight specimen (Figure 3), and in this instance the shear force V was applied at the location of the roller supports.

When using boundary collocation, convergence of the solution must be demonstrated. Convergence problems arise at extremes in the parameters a/W, r_1/r_2, and S/W. Solutions were obtained using $r_1/r_2 = 0.6$, $S/W = 4$, and $a/W =$ 0.2 or 0.8. A large number of equally-spaced boundary stations (m) were established and the number of coefficients (2N) in the stress function was systematically increased. Adequate convergence (less than 0.1 percent) of both K and crack mouth displacements was obtained when $m = 120$, $2N = 60$, and $a/W =$ 0.2. At the other extreme, convergence was not obtained even when $m = 240$ and

2N = 160. In the interest of prudent computer usage, it was decided to use m = 120 and 2N = 60 and limit the reported solutions to crack lengths whose accuracy can be assured by approaching the appropriate limiting solutions.

RESULTS

Normalized stress intensity factors appear in Table I, normalized crack mouth displacements are in Table II, and normalized load-line displacements are in Table III. The published solutions for the straight geometry are also given in each table.

TABLE I. NORMALIZED STRESS INTENSITY FACTORS

$$\frac{KB\sqrt{W}}{P}$$

S/W = 4

r_1/r_2	a/W			
	0.2	0.3	0.4	0.5
1.0 (ref 8)	4.70	6.08	7.93	10.65
1.0 (this study)	4.66	6.07	7.91	10.65
0.8	4.86	6.23	8.03	10.77
0.7	4.94	6.28	8.11	10.88
0.6	5.03	6.39	8.20	10.88

S/W = 3

r_1/r_2	a/W				
	0.2	0.3	0.4	0.5	0.6
1.0 (ref 8)	3.52	4.56	5.95	7.99	11.31
1.0 (this study)	3.40	4.43	5.78	7.76	11.06
0.8	3.50	4.53	5.87	7.86	11.16
0.6	3.62	4.63	5.98	7.98	11.28
0.4	3.67	4.64	6.07	8.09	11.33

TABLE II. NORMALIZED TOTAL CRACK MOUTH OPENING DISPLACEMENTS

$$\frac{EB\delta_{cm}}{P}$$

S/W = 4

r_1/r_2	a/W			
	0.2	0.3	0.4	0.5
1.0 (ref 9)	7.07	12.36	20.83	35.67
1.0 (this study)	6.83	12.22	20.76	36.75
0.8	7.47	12.64	21.28	36.09
0.7	7.54	12.89	21.69	36.78
0.6	7.72	13.26	22.41	37.75

S/W = 3

r_1/r_2	a/W				
	0.2	0.3	0.4	0.5	0.6
1.0 (ref 9)	5.31	9.27	15.62	26.75	48.27
1.0 (this study)	5.04	9.09	15.32	26.07	47.00
0.8	5.19	9.01	15.64	26.62	47.85
0.6	5.53	9.78	16.14	27.31	49.01
0.4	5.78	10.01	16.34	27.76	50.20

TABLE III. NORMALIZED LOAD-LINE DISPLACEMENTS

$$\frac{EB\delta_{LL}}{P}$$

S/W = 4

r_1/r_2	a/W 0.2	0.3	0.4	0.5
1.0 (ref 10)	23.44	28.74	37.05	53.46
1.0 (this study)	23.83	29.38	39.09	55.46
0.8	23.61	29.93	39.74	57.28
0.7	24.16	30.43	40.75	59.77
0.6	23.59	31.37	41.83	59.81

S/W = 3

r_1/r_2	a/W 0.2	0.3	0.4	0.5	0.6
1.0 (ref 10)	11.52	14.50	19.49	28.39	45.51
1.0 (this study)	11.64	15.13	20.36	29.18	46.38
0.8	11.87	15.40	20.34	29.15	47.28
0.6	12.89	16.51	21.39	30.93	49.12
0.4	10.84	16.96	17.97	33.54	47.57

A discussion of these results yields a few general findings. The straight three-point bend solutions generated here for S/W = 4 agree well with previously published data (refs 8-10) with the exception of the load-line displacements where disagreement of as much as 4 percent is observed at a/W = 0.5. This may be because the total deflection given by Haggag and Underwood (ref 10) is a constant term (deflection due to shear and bending with no crack) plus a term that varies with crack length, the latter being taken from Tada (ref 9). Recent

work (ref 11) has shown that integration of the K solution using Irwin's equation for the three-point bend sample does not agree well with the deflections reported in References 9 and 10. The results published by Underwood et al. (ref 11) nominally agree within 1 percent with the results reported here.

The S/W = 3 solutions for the straight specimen show a consistent trend with published results. The collocation solutions are always somewhat less than the values given in References 8, 9, and 10, again with the exception of the load-line deflection. This can be explained by the realization that the values given in the first row of each table were calculated from wide range expressions that assume a given S/W dependence, and were fit to results for the specific case of S/W = 4. Since this is the first time to the author's knowledge that collocation has been used for the case of S/W = 3, and the S/W = 4 solutions agree very well with the previous solutions, it is safe to assume that the collocation solutions reported here for S/W = 3 are as accurate as the S/W = 4 solutions. This suggests that the effect of S/W is not exactly taken care of with the wide range expressions given in References 8, 9, and 10. It also implies that wide range expressions must account for more than the simple S/W dependence that is derived from limiting solutions or that separate wide range expressions should be developed for each S/W configuration.

The effect of radius ratio (r_1/r_2) on the obtained results is as expected from previous results (ref 6). As r_1/r_2 decreases, the geometry becomes more of a curved beam, and stresses at r_1 become higher. This results in an elevation in K, and correspondingly, with crack mouth opening and load-line displacement when the cracks are shallow relative to straight beam specimens. As the crack length increases, the effect of curvature should diminish, and in the limit when the crack approaches through thickness, it disappears. This is the case for all

the solutions with the only exception being the load-line displacement of the very thick cylinder ($r_1/r_2 = 0.4$). In this case, the chord length $2r_2 \sin \theta_o$ was unable to exceed S by 10 percent. This probably resulted in some end effects on the displacement of the roller support location. This could not be avoided using the method of solution outlined above. Therefore, the load-line displacement of the thick cylinder ($r_1/r_2 = 0.4$) should be considered suspect. Nevertheless, this end effect appears only in the load-line displacement. It is clear that the proper, consistent trend in both K and crack mouth opening is occurring, and these solutions, even for the thick cylinder, should be considered accurate.

WIDE RANGE EXPRESSIONS

The technique of fitting wide range expressions to numerical data has been outlined by Srawley (ref 12) for K solutions and by Saxena and Hudak (ref 13) for displacements. Basically, the method is to find a nondimensional form of K or displacement that has finite limits as a/W approaches both zero and one. For K, the form to use is the following (ref 12):

$$\frac{KB\sqrt{W}}{P(S/W)} \frac{(1-a/W)^{3/2}}{\sqrt{a/W}} = g_K(r_1/r_2, a/W) \cdot f_K(a/W) \tag{22}$$

$$\begin{aligned} \lim_{a/W \to 1} g_K \cdot f_K &= 0.995 \\ \lim_{\substack{a/W \to 0 \\ r_1/r_2 \to 1}} g_K \cdot f_K &= 2.987 \end{aligned} \tag{23}$$

The function $g_K(r_1/r_2, a/W)$ is included to account for effects of r_1/r_2. The nondimensional form given in Eq. (22) is plotted in Figure 4 for S/W = 4 and in Figure 5 for S/W = 3. It is clear that this representation does not account totally for the effect of S/W, therefore, it was decided to fit two different expressions, one each for S/W = 3 and S/W = 4.

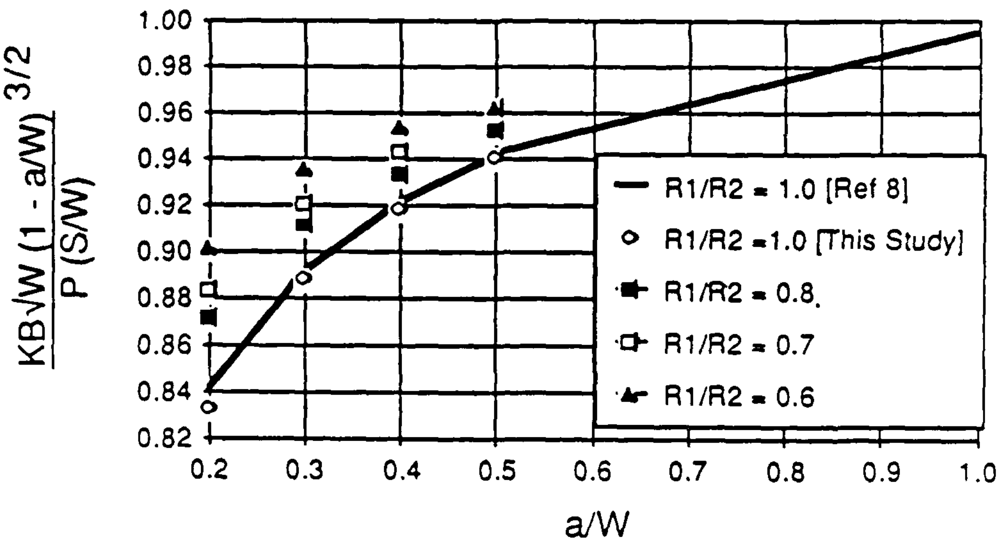

Figure 4. Normalized stress intensity factor solution; S/W = 4.

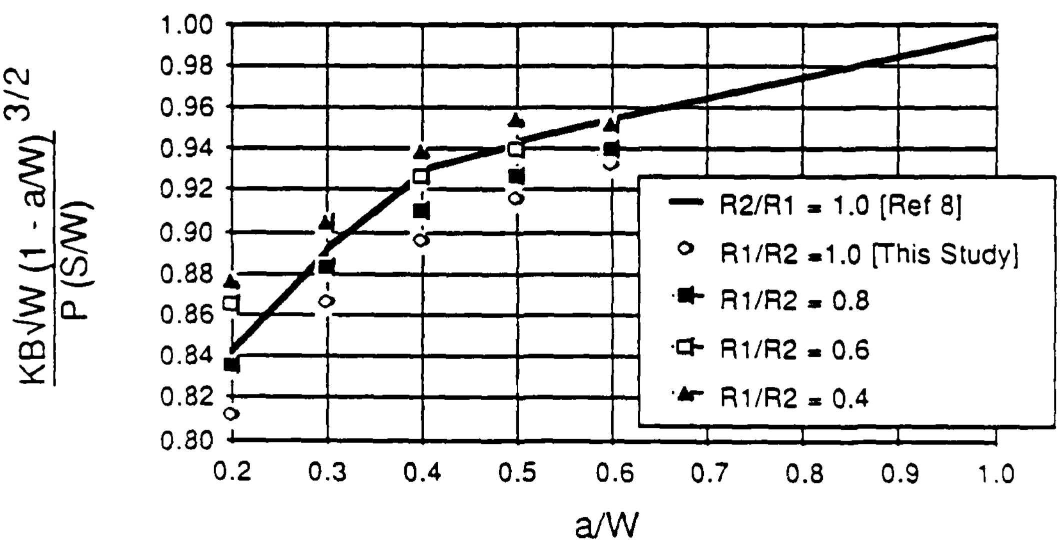

Figure 5. Normalized stress intensity factor solution; S/W = 3.

The function g_K was found by using the limiting solution ($r_1/r_2 = 1$) as a reference K solution. By taking the ratio of $KB\sqrt{W}(1-a/W)^{3/2}/(P(S/W)\sqrt{a/W})$ of the curved specimen to the straight specimen at constant a/W values, g_K can be easily constructed over a range of r_1/r_2 and a/W. The constructed or calculated g_K can then be easily fit using multivariable least squares. Once g_K was found, it was factored out and $f_K(a/W)$ was also found by least squares. For either fit, it was assumed that g_K and f_K were both polynomials. The following are the results of the fitting:

For S/W = 4:

$$\frac{KB\sqrt{W}}{P(S/W)} \frac{(1-a/W)^{3/2}}{\sqrt{a/W}} = (1+(1-r_1/r_2)(0.34 - 0.82\ a/W + 0.48(a/W)^2)) \cdot F(a/W)$$

$$F(a/W) = 2.65 - 5.07(a/W) + 6.24(a/W)^2 - 2.93(a/W)^3 \tag{24}$$

For S/W = 3:

$$\frac{KB\sqrt{W}}{P(S/W)} \frac{(1-a/W)^{3/2}}{\sqrt{a/W}} = (1+(1-r_1/r_2)(0.21 - 0.40\ a/W + 0.19(a/W)^2)) \cdot F(a/W)$$

$$F(a/W) = 2.517 - 4.431\ a/W + 5.028(a/W)^2 - 2.120(a/W)^3 \tag{25}$$

The agreement between Eqs. (24) and (25) and the collocation data are shown in Tables IV and V. From this comparison, the following accuracy statements can be made. For S/W = 4, Eq. (24) agrees with collocation results within ± 0.5 percent for $r_1/r_2 \geq 0.6$ and $a/W \geq 0.2$. For S/W = 3, Eq. (25) agrees with collocation results within 1.0 percent for $r_1/r_2 \geq 0.4$ and $a/W \geq 0.2$ and within ± 0.8 percent for $r_1/r_2 \geq 0.4$ and $0.4 \leq a/W \leq 0.6$.

For crack mouth opening displacements, a similar procedure can be followed, but it is necessary to use different limits. Briefly, it can be shown that for crack mouth opening the following nondimensional form is appropriate (ref 13):

$$\frac{EB\delta_{cm}}{P(S/W)} \frac{(1-a/W)^2}{(a/W)} = g_{cm}(r_1/r_2, a/W) \cdot f_{cm}(a/W) \tag{26}$$

$$\lim_{a/W \to 1} g_{cm} \cdot f_{cm} = 3.95 \tag{27}$$

$$\lim_{\substack{a/W \to 1 \\ r_1/r_2 \to 1}} g_{cm} \cdot f_{cm} = 8.736 \tag{28}$$

TABLE IV. NORMALIZED STRESS INTENSITY FACTORS (S/W = 4)

$$\frac{KB\sqrt{W}\,(1-a/W)^{1.5}}{P(S/W)\sqrt{a/W}}$$

r_1/r_2		a/W					
		0.0	0.2	0.3	0.4	0.5	1.0
	Collocation	2.987	1.863	1.623	1.435	1.331	0.994
1.0	Equation 24	2.650	1.866	1.620	1.449	1.334	0.990
	Relative Err	-0.113	0.002	-0.002	-0.003	0.002	-0.004
	Collocation		1.946	1.664	1.475	1.346	0.994
0.8	Equation 24		1.939	1.665	1.475	1.347	0.990
	Relative Err		-0.004	0.001	-	0.001	-0.004
	Collocation		1.974	1.680	1.490	1.360	0.994
0.7	Equation 24		1.975	1.687	1.487	1.354	0.990
	Relative Err		0.001	0.004	-0.002	-0.005	-0.004
	Collocation		2.013	1.708	1.507	1.360	0.994
0.6	Equation 24		2.012	1.709	1.500	1.360	0.990
	Relative Err		-0.001	0.001	-0.004	-	-0.004

TABLE V. NORMALIZED STRESS INTENSITY FACTORS (S/W = 3)

$$\frac{KB\sqrt{W}\ (1-a/W)^{1.5}}{P(S/W)\sqrt{a/W}}$$

r_1/r_2		a/W						
		0.0	0.2	0.3	0.4	0.5	0.6	1.0
	Collocation	2.987	1.815	1.580	1.416	1.296	1.204	0.994
1.0	Equation 25	2.517	1.815	1.583	1.413	1.294	1.211	0.990
	Relative Err	-0.157	-	0.002	-0.002	-0.002	0.005	-
	Collocation		1.867	1.611	1.439	1.311	1.214	0.994
0.8	Equation 25		1.865	1.617	1.436	1.308	1.220	0.994
	Relative Err		-0.001	0.002	-0.002	-0.002	0.005	-
	Collocation		1.935	1.652	1.465	1.330	1.228	0.994
0.6	Equation 25		1.915	1.651	1.459	1.323	1.229	0.994
	Relative Err		-0.010	-0.001	-0.004	-0.005	0.001	-
	Collocation		1.958	1.652	1.485	1.349	1.234	0.994
0.4	Equation 25		1.965	1.685	1.482	1.338	1.238	0.994
	Relative Err		0.003	0.020	-0.002	-0.008	0.004	-

The collocation results using the nondimensional form of Eq. (26) are plotted in Figures 6 and 7 for S/W = 4 and S/W = 3, respectively. Using the procedure outlined above, the wide range expressions for crack mouth opening are the following:

For S/W = 4:

$$\frac{EB\delta_{cm}}{P(S/W)}\ \frac{(1-a/W)^2}{(a/W)} = (1+(1-r_1/r_2)(0.70 - 1.94\ a/W + 1.23(a/W)^2)) \cdot F(a/W)$$

$$F(a/W) = 7.36 - 13.16\ a/W + 20.62(a/W)^2 - 10.87(a/W)^3 \qquad (29)$$

For S/W = 3:

$$\frac{EB\delta_{cm}}{P(S/W)} \, \frac{(1-a/W)^2}{(a/W)} = (1+(1-r_1/r_2)(0.21 - 0.10\, a/W + 0.11(a/W)^2 - 2.61(a/W)^3))$$

$$\cdot \; F(a/W)$$

$$F(a/W) = 7.09 - 10.66\, a/W + 12.88(a/W)^2 - 5.37(a/W)^3 + 3.76(a/W)^4 \qquad (30)$$

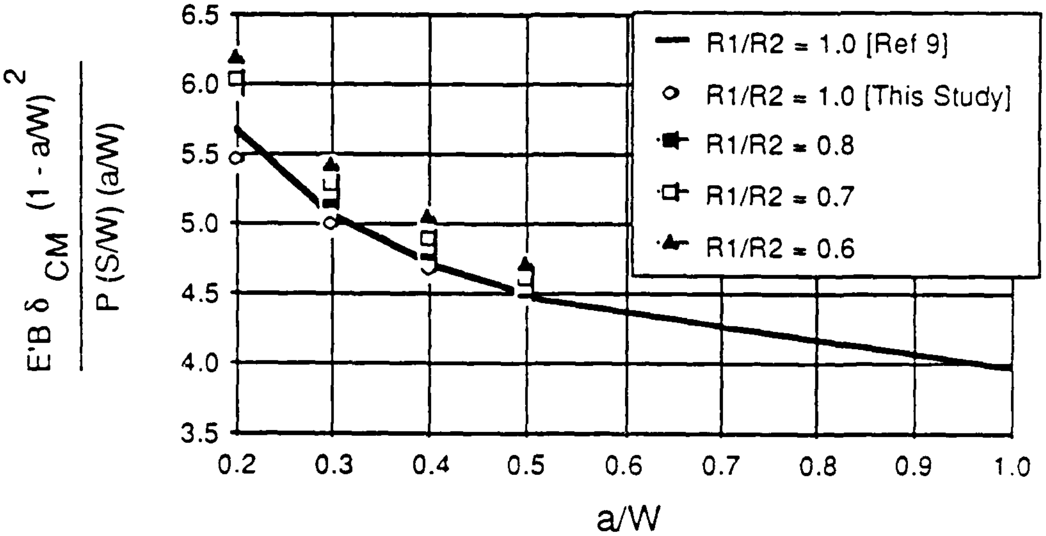

Figure 6. Normalized crack mouth opening displacement solution; S/W = 4.

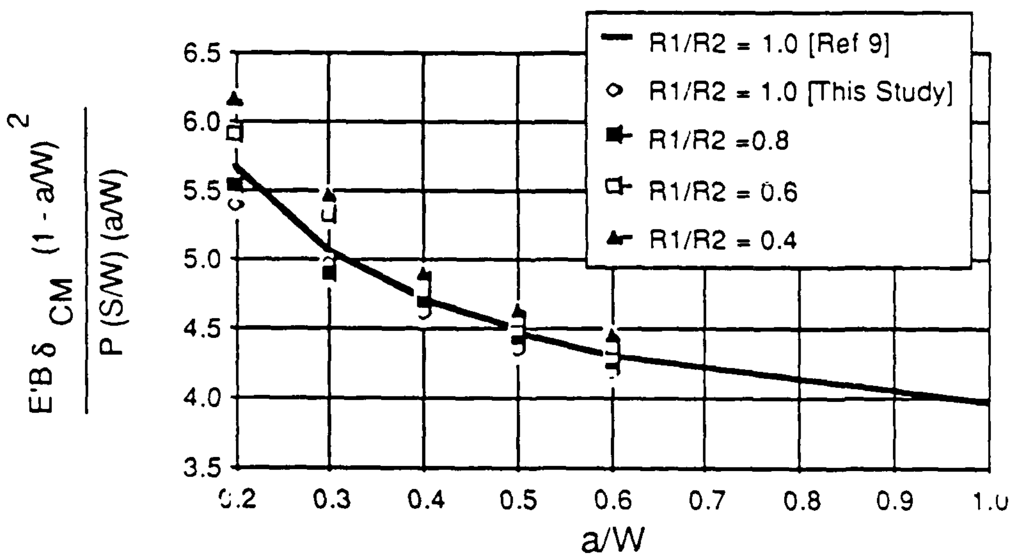

Figure 7. Normalized crack mouth opening displacement solution; S/W = 3.

Comparisons between the collocation results and Eqs. (29) and (30) are given in Tables VI and VII. The following accuracy statements can be made. For $S/W = 4$, Eq. (29) agrees with collocation results within ± 2.5 percent for $r_1/r_2 \geqslant 0.6$ and $a/W \geqslant 0.2$. For $S/W = 3$, Eq. (30) agrees with collocation results within ± 2.5 percent for $r_1/r_2 \geqslant 0.4$ and $a/W \geqslant 0.2$ and within ± 1.5 percent for $r_1/r_2 \geqslant 0.4$ and $0.4 \leqslant a/W \leqslant 0.6$.

TABLE VI. NORMALIZED CRACK MOUTH OPENING DISPLACEMENTS (S/W = 4)

$$\frac{E'B\delta_{cm}(1-a/W)^2}{P(S/W)(a/W)}$$

r_1/r_2		a/W					
		0.0	0.2	0.3	0.4	0.5	1.0
	Collocation	8.736	5.460	4.991	4.672	4.594	3.950
1.0	Equation 29	7.360	5.466	4.974	4.700	4.576	3.950
	Relative Err	-0.158	0.001	-0.003	0.006	-0.004	-
	Collocation		5.973	5.161	4.787	4.511	3.950
0.8	Equation 29		5.861	5.202	4.813	4.611	3.942
	Relative Err		-0.019	0.008	0.005	0.022	-0.002
	Collocation		6.031	5.264	4.881	4.597	3.950
0.7	Equation 29		6.058	5.316	4.870	4.628	3.938
	Relative Err		0.004	0.010	-0.002	0.007	-0.003
	Collocation		6.181	5.419	5.043	4.719	3.950
0.6	Equation 29		6.256	5.429	4.927	4.645	3.934
	Relative Err		0.012	0.002	-0.023	-0.016	-0.004

TABLE VII. NORMALIZED CRACK MOUTH OPENING DISPLACEMENTS (S/W = 3)

$$\frac{E'B\delta_{cm}(1-a/W)^2}{P(S/W)(a/W)}$$

r_1/r_2		a/W						
		0.0	0.2	0.3	0.4	0.5	0.6	1.0
	Collocation	8.736	5.374	4.950	4.596	4.435	4.177	3.950
1.0	Equation 30	7.090	5.430	4.906	4.543	4.309	4.171	3.940
	Relative Err	-0.189	0.010	-0.009	-0.012	-0.008	-0.001	-0.003
	Collocation		5.532	4.904	4.693	4.436	4.254	3.950
0.8	Equation 30		5.632	5.073	4.682	4.423	4.263	3.940
	Relative Err		0.018	0.034	-0.002	-0.003	0.002	-0.003
	Collocation		5.901	5.323	4.842	4.552	4.357	3.950
0.6	Equation 30		5.833	5.230	4.820	4.537	4.355	3.940
	Relative Err		-0.011	-0.016	-0.005	-0.003	-	-0.003
	Collocation		6.163	5.451	4.903	4.627	4.463	3.950
0.4	Equation 30		6.035	5.407	4.959	4.651	4.447	3.940
	Relative Err		-0.021	-0.008	0.011	0.005	-0.004	-0.003

For total load-line displacements, the following is the appropriate normalized form (ref 11):

$$\frac{EB\delta_{LL}(1-a/W)^2}{P(S/W)} = g_{LL}(r_1/r_2, a/W) \cdot f_{LL}(a/W) \qquad (31)$$

$$\lim_{a/W\to 1} g_{LL} \cdot f_{LL} = 0.9875 \qquad (32)$$

$$\lim_{\substack{a/W\to 0\\ r_1/r_2\to 1}} g_{LL} \cdot f_{LL} = 1.195 \ (S/W = 4)$$

$$= 1.010 \ (S/W = 3) \qquad (33)$$

The collocation results normalized according to Eq. (31) are plotted in Figure 8 for S/W = 4 and in Figure 9 for S/W = 3. Since we have found that the

load-line displacements are not as accurate as the stress intensity factors or the crack mouth opening displacements, it was decided to forego fitting wide range expressions to these numerical results.

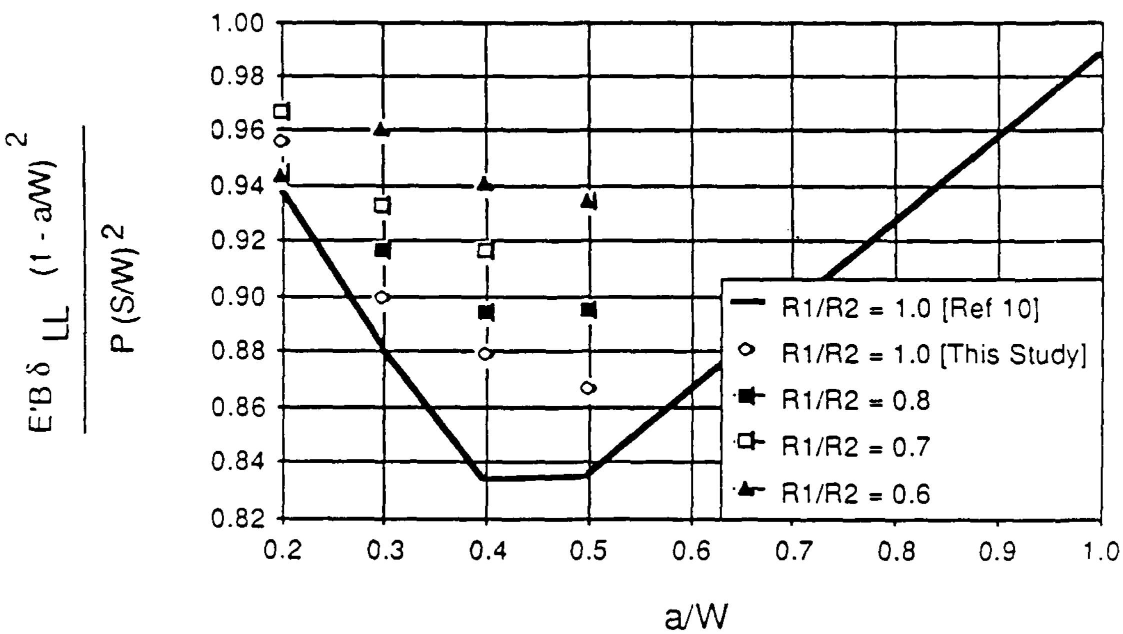

Figure 8. Normalized load-line displacement solution; S/W = 4.

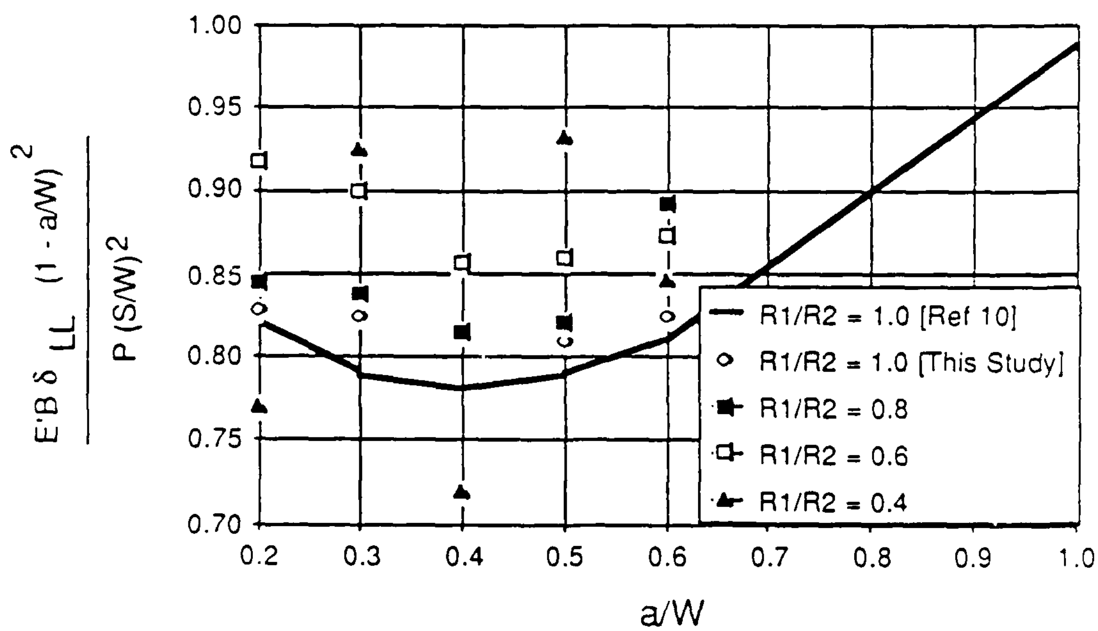

Figure 9. Normalized load-line displacement solution; S/W = 3.

SUMMARY AND CONCLUSIONS

The arc bend-chord support specimen (AB(C)) has been studied using collocation over a wide range of possible testing geometries. Stress intensity factors, crack mouth opening displacements, and total load-line deflections were obtained from the analysis. Wide range equations were fit to the stress intensity factor and crack mouth opening displacement solutions to make the solutions more suitable for possible inclusion in future drafts of E-399 on Plane-Strain Fracture Toughness of Metallic Materials.

REFERENCES

1. Underwood, J. H., Kapp, J. A., and Witherell, M. D., "Fracture Testing Using Arc Bend Samples," Fracture Mechanics: Seventeenth Volume, ASTM STP 905, (J. H. Underwood, et al., eds.), American Society for Testing and Materials, Philadelphia, PA, 1986, pp. 279-296 (also ARLCB-TR-85014, U.S. Army ARDC, Benet Weapons Laboratory, Watervliet, NY, May 1985).

2. Gross, B. and Mendelson, A., "Plane Elastostatic Analyses of V-Notched Plates," International Journal of Fracture Mechanics, Vol. 8, No. 3, September 1973, pp. 267-276.

3. Gross, B., Roberts, E. J., and Srawley, J. E., "Elastic Displacements for Various Edge-Cracked Plate Specimens," International Journal of Fracture Mechanics, Vol. 4, No. 3, September 1968, pp. 267-276 (Errata, Vol. 6, 1970, p. 84).

4. Hussain, M. A., Lorensen, W. E., Kendall, D. P., and Pu, S. L., "A Modified Collocation Method for C-Shaped Specimens," Benet Weapons Laboratory Technical Report R-WV-T-X-6-73, Watervliet, NY, February 1973.

5. Williams, M. L., "On the Stress Distribution at the Base of a Stationary Crack," Journal of Applied Mechanics, Vol. 24, No. 1, March 1957, pp. 109-114.

6. Gross, B. and Srawley, J. E., "Analysis of Radically Cracked Ring Segments Subject to Forces and Couples," Development in Fracture Mechanics Test Methods Standardization, ASTM STP 632, American Society for Testing and Materials, Philadelphia, PA, 1977, pp. 39-56.

7. Seely, F. B. and Smith, J. D., Advanced Mechanics of Materials, Second Edition, John Wiley & Sons, Inc., New York, 1952.

8. Gross, B. and Srawley, J. E., "Stress Intensity Factors for Single-Edge Notch Specimens in Bending or Combined Bending and Tension by Boundary Collocation of a Stress Function," NASA TND-2603, National Aeronautics and Space Administration, Washington, D.C., January 1965.

9. Tada, H., Paris, P., and Irwin, G., The Stress Analysis of Cracks Handbook, Del Research Corporation, Hellertown, PA, 1973.

10. Haggag, F. M. and Underwood, J. H., "Compliance of a Three-Point Bend Specimen at Load Line," International Journal of Fracture, Vol. 26, October 1984, pp. R63-R65 (also ARLCB-MR-84035, U.S. Army ARDC, Benet Weapons Laboratory, Watervliet, NY, October 1984).

11. Underwood, J. H., Kapp, J. A., and Baratta, F. I., "More on Compliance of the Three-Point Bend Specimen," International Journal of Fracture, Vol. 28, No. 2, June 1985, pp. R41-R45 (also ARLCB-MR-85015, U.S. Army ARDC, Benet Weapons Laboratory, Watervliet, NY, May 1985).

12. Srawley, J. E., "Wide Range Stress Intensity Factor Expressions for ASTM E-399 Standard Fracture Toughness Specimens," International Journal of Fracture, Vol. 12, No. 3, June 1976, pp. 475-476.

13. Saxena, A. and Hudak, S. J. Jr., "Review and Extension of Compliance Information for Common Crack Growth Specimens," International Journal of Fracture, Vol. 14, No. 5, October 1978, pp. 453-468.

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